filter designs, the same gain for both stop-bands), and the pass-band will be scaled by a gain factor, denoted G_{pass}, which would ideally be equal to unity. (By convention, all gain factors are expressed in decibels defined as: $G_{dB} = 20 \log_{10} G$.)

A nonlinear filter is a signal-processing device whose output is not a linear function of its input. Terminology concerning the filtering problem may refer to the time domain (state space) representation of the signal or to the frequency domain representation of the signal. When referring to filters with adjectives such as "band pass, high pass, and low pass" one has in mind the frequency domain. When resorting to terms like "additive noise", one has in mind the time domain, since the noise that is to be added to the signal is added in the state space representation of the signal. The state space representation is more general and is used for the advanced formulation of the filtering problem as a mathematical problem in probability and statistics of stochastic processes.

2.2. Median Filter

A median filter, which has many applications in ECG processing and it is a favourite tool among ECG society for smoothing filtering. The best known feature of median filters is their ability to remove noise while retaining sudden changes in the signal. The median filter is implemented by sliding a window of odd length over the signal one sample at time. At each window position the samples inside the filter window are sorted by magnitude and the mid-value (median) is the filter output. We denote the filter length N and since required to be odd it can be represented as N = 2k +1. The output of the filter is then the (k+1)th largest or smallest sample in the filter window. The filtering procedure is denoted as:

$$y(n) = median[x(n-k), \dots, x(n), \dots, x(n+k)] \qquad (1)$$

Where *x (n)* and *y (n)* is the input and output sequences. If the samples in the filter window are denoted as $x_1, x_2, x_3 \dots\dots\dots x_{2k+1}$ notation that is commonly used from the sorted list of those samples is $x_{(1)}, x_{(2)}, \dots\dots\dots x_{(2k+1)}$. Here, $x_{(1)}$, $x_{(2k+1)}$ are the smallest and the greatest sample, respectively. By using this notation the median value is $x_{(k+1)}$. The definition of median for an even and odd number of samples is given by:

$$med(x_1, \dots, x_N) = \begin{cases} x_{(k+l)} & N = 2K+1 \\ \frac{1}{2} x_k + x_{(k+l)} & N = 2K \end{cases} \qquad (2)$$

When the filter window is centered at the beginning or at the end of the input signal some value must be assigned to the empty window position. The first and the last value carry-on appending strategy are used. This means that values from *x (-k)* to *x (-1)* are taken to be equal to *x(0)*, and the values from *x(L)* to *x(L+k-1)* are equal to *x(L-1)*, where the signal is consisted of samples from *x(0)* to *x(L-1)*. One of the major problems with the median filter is that it is relatively expensive and complex to compute. To find the median it is necessary to sort all the values in the neighbourhood into numerical order and this is relatively slow [8]-[9].

2.3. Moving Average Filter

The moving average is the most common filter in DSP, mainly because it is the easiest digital filter to understand and use. In spite of its simplicity, the moving average filter is *optimal* for a common task: reducing random noise. As the name implies, the moving average filter operates by averaging a number of points from the input signal to produce each point in the output signal. In equation form, this is written:

$$y[i] = \frac{1}{M} \sum_{j=0}^{M-1} x(i+j)$$

Equation of the moving average filter. In this equation, *x* [ ] is the input signal, *y* [ ] is the output signal, and *M* is the number of points used in the moving average. This equation only uses points on *one side* of the output sample being calculated. In a 5 point moving average filter, point 80 in the output signal is given by:

$$y[80] = \frac{x[80]+x[81]+x[82]+x[83]+x[84]}{5}$$

As an alternative, the group of points from the input signal can be chosen *symmetrically* around the output point:

$$y[80] = \frac{x[78]+x[79]+x[80]+x[81]+x[82]}{5}$$

This corresponds to changing the summation in first equation from: *j*= 0 to *M*-1, to: *j*=*0*-(*M*-1) /2 to (*M*+1) /2. For instance, in a 10 point moving average filter, the index, *j*, can run from 0 to 11 (one side averaging) or -5 to 5 (symmetrical averaging). Symmetrical averaging requires that *M* be an *odd* number. Programming is slightly easier with the points on only one side; however, this produces a relative shift between the input and output signals. You should recognize that the moving average filter is a *convolution* using a very simple filter kernel. For example, a 5 point filter has the filter kernel: þ 0, 0, 1/5, 1/5, 1/5, 1/5, 1/5, 0, 0 þ. That is, the moving average filter is a convolution of the input signal with a rectangular pulse having an area of one [17].

2.4.Kalman Filter

The Kalman filter is an efficient recursive filter that estimates the state of a linear dynamic system from a series of noisy measurements. It is used in a wide range of engineering applications from radar to computer vision, and is an important topic in control theory and control systems engineering. Together with the linear-quadratic regulator (LQR), the Kalman filter solves the linear-quadratic-Gaussian control problem (LQG). The Kalman filter, the linear-quadratic regulator and the linear-quadratic-Gaussian controller are solutions to what probably are the most fundamental problems in control theory.